GCE

in

Mathematics

S P E C I F I C A T I O N

(Amended August 2010)

For teaching from **September 2010**

for examination in **2011**

FOREWORD

This booklet contains CCEA's Advanced Subsidiary (AS) and Advanced (A level) GCE Mathematics specification for teaching from September 2004. This specification has been developed to take account of the revised AS and A level GCE Mathematics Subject Criteria developed jointly by the Regulatory Authorities in England (QCA), Northern Ireland (CCEA) and Wales (ACCAC) and published by QCA in December 2002.

The AS is the first half of the A level and will be assessed at a standard appropriate for candidates who have completed half of the A level course. The A level course comprises the AS together with the second half of the A level course, referred to as A2. A2 will be assessed at a standard appropriate for candidates who have completed the full A level course and will contain an element of synoptic assessment. The A level award will be based on the aggregation of marks from the AS (50%) and A2 (50%).

AS can be taken as a 'stand alone' qualification without progress to A level.

The first year of certification of the AS and A Level is 2005.

Contents

KEY FEATURES

This suite of syllabuses adheres to the GCE Advanced Subsidiary (AS) and Advanced (A Level) Subject Criteria for Mathematics (2002).

The syllabuses:

- seek to consolidate and extend the knowledge, skills and understanding developed in Key Stage 4;
- have a structure which will allow candidates of all abilities to have the opportunity to demonstrate positive achievement;
- provide a suitable foundation for study of mathematics and other subjects in further and higher education and for a range of interesting careers;
- enable schools and colleges to provide a coherent, satisfying and worthwhile course of study for students who do not progress to further study of mathematics;
- give centres the flexibility to decide which scheme of assessment best caters for the needs of their students.

SUMMARY OF EXAMINATION INFORMATION

The specification adopts a modular structure. Candidates are required to study three teaching and learning modules for the AS and six modules for the A level GCE in Mathematics and in Further Mathematics.

Teaching and learning module	Assessment Unit	Nature of Assessment	Assessment weighting (%)	Examination Session Availability
Module C1: AS Core Mathematics 1	C1 Assessed at AS level. Compulsory for AS and A level in Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module C2: AS Core Mathematics 2	C2 Assessed at AS level. Compulsory for AS and A level in Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module C3: A2 Core Mathematics 1	C3 Assessed at A2 level. Compulsory for A level in Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	16 ⅔ % of A Level	Summer and Winter
Module C4: A2 Core Mathematics 2	C4 Assessed at A2 level. Compulsory for A level in Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	16 ⅔ % of A Level	Summer and Winter
Module FP1: Further Pure Mathematics 1	FP1 Assessed at AS level. Compulsory for AS and A level in Further Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module FP2: Further Pure Mathematics 2	FP2 Assessed at A2 level. Compulsory for A level in Further Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module FP3: Further Pure Mathematics 3	FP3 Assessed at A2 level. Compulsory for A Level in Further Mathematics.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer only

Teaching and learning module	Assessment Unit	Nature of Assessment	Assessment weighting (%)	Examination Session Availability
Module M1: Mechanics 1	M1 Assessed at AS level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module M2: Mechanics 2	M2 Assessed at A2 level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module M3: Mechanics 3	M3 Assessed at A2 level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer only
Module M4: Mechanics 4	M4 Assessed at A2 level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	16 ⅔ % of A Level	Summer only
Module S1: Statistics 1	S1 Assessed at AS level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer and Winter
Module S2: Statistics 2	S4 Assessed at A2 level. Optional.	1 hour 30 minutes external examination paper. Maximum 75 raw marks. 6 – 8 questions.	33 ⅓ % of AS 16 ⅔ % of A Level	Summer only

NOTE: The Assessment Unit for Module S2 is designated S4

1 INTRODUCTION

RATIONALE

Mathematics is inherently a sequential subject. There is a progression of material through all levels at which the subject is studied. The content, therefore, builds upon the knowledge, skills and understanding established at GCSE. The core content for AS is a subset of the core content for A level. Progression in the subject will extend in a natural way beyond AS and A level, into Further Mathematics or into related courses in higher education.

This specification adheres to the 2002 Subject Criteria for AS and A level Mathematics and has been designed to conform with the GCE Advanced Subsidiary and Advanced Level Examinations Qualification-Specific Criteria and Common Criteria established jointly by the regulatory authorities in England, Wales and Northern Ireland and published by the Qualifications and Curriculum Authority (QCA).

In following a course based on this specification students should have opportunities to:

- consolidate and extend the knowledge, skills and understanding developed in Key Stage 4;
- demonstrate positive achievement;
- build a suitable foundation for study of mathematics and other subjects in further and higher education;
- prepare themselves for their economic environment and for a range of interesting careers;
- enjoy a coherent, satisfying and worthwhile course of study.

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced.

Where appropriate, teachers should make opportunities to address spiritual, moral, ethical, social and cultural issues and promote an awareness of environmental, health and safety and European issues and developments. For example, students taking a statistics module may be given the opportunity to discuss presentation of data and the possible misrepresentation of information to support a particular point of view. These issues will not be directly assessed in any of the assessment units.

This specification has been designed to be as free as possible from ethnic, gender, religious, political or other forms of bias.

AIMS

Courses based on this specification should encourage students to:

a develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;

b develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;

c extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;

d develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;

e recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;

f use mathematics as an effective means of communication;

g read and comprehend mathematical arguments and articles concerning applications of mathematics;

h acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;

i develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;

j take increasing responsibility for their own learning and the evaluation of their own mathematical development.

ASSESSMENT OBJECTIVES

The assessment objectives provide an indication of the skills and abilities which the assessment units are designed to assess, together with the knowledge and understanding specified in the subject content. It is not always possible to make a clear distinction between these different elements in constructing examination questions and therefore a particular question may test more than one assessment objective.

The assessment objectives and the associated weightings for AS and A level are the same and are listed below.

Candidates should be able to:

Assessment Objectives		Minimum Weighting
AO1	recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts;	30%
AO2	construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form;	30%
AO3	recall, select and use their knowledge of standard mathematical models to represent situations in the real world; recognise and understand given representations involving standard models; present and interpret results from such models in terms of the original situation, including discussion of the assumptions made and refinement of such models;	10%
AO4	comprehend translations of common realistic contexts into mathematics; use the results of calculations to make predictions, or comment on the context; and, where appropriate, read critically and comprehend longer mathematical arguments or examples of applications;	5%
AO5	use contemporary calculator technology and other permitted resources (such as formulae booklets or statistical tables) accurately and efficiently; understand when not to use such technology, and its limitations; give answers to appropriate accuracy.	5%

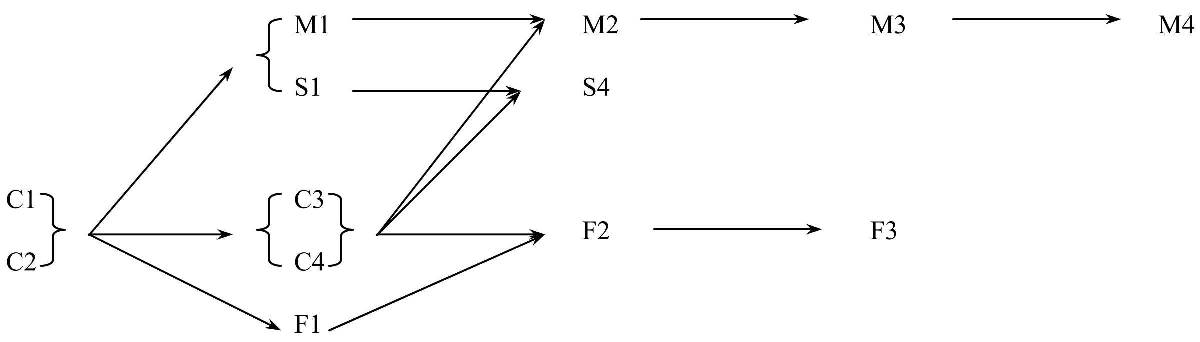
Not all assessment objectives will be assessed in every paper but the total assessment will examine the assessment objectives as set out above. The weightings given to any particular objective may vary from year to year but will reflect the minimum requirements indicated above. The weighting given to the assessment objectives in each paper will reflect the principles of fitness for purpose and will take into account the nature of the module being assessed.

SPECIFICATION STRUCTURE

The specification adopts a modular structure. Candidates are required to take three assessment units for an award in AS Mathematics and six assessment units for an award in A level Mathematics. The requirements are the same for the corresponding awards in Further mathematics. The available assessment units are listed below:

Assessment Unit	**Standard**	**Requirement**
C1	AS	Compulsory for AS and A level Mathematics
C2	AS	Compulsory for AS and A level Mathematics
C3	A2	Compulsory for A level Mathematics
C4	A2	Compulsory for A level Mathematics
F1	AS	Compulsory for AS and A level Further Mathematics
F2	A2	Compulsory for A level Further Mathematics
F3	A2	Compulsory for A level Further Mathematics
M1	AS	Optional
M2	A2	Optional
M3	A2	Optional
M4	A2	Optional
S1	AS	Optional
S4	A2	Optional

The diagram below outlines the dependence of each assessment unit on others in the specification:



KEY SKILLS

This specification provides opportunities for developing and generating evidence for assessing the following nationally specified key skills at the levels indicated :

- Application of Number Level 3
- Communication Level 3
- Information Technology Level 3
- Working with Others Level 3
- Improving Your Own Learning and Performance Level 3
- Problem Solving Level 3

The opportunities provided are referenced to the relevant key skills specifications and exemplified in the Appendix on page 64.

PROHIBITED COMBINATIONS

In any one series of examinations a candidate may not take examinations on this specification together with examinations on another specification of the same title.

It is not permitted to count the award from the same module:

- in two Advanced GCE subjects;
- in two AS subjects.

Every specification is assigned to a national classification code indicating the subject area to which it belongs.

Centres should be aware that candidates who enter for more than one GCE qualification with the same classification code, will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification codes for this specification are:

2211 Mathematics

2231 Pure Mathematics

2331 Further Mathematics

2 SCHEME OF ASSESSMENT

ALLOWABLE COMBINATIONS

From the assessment units listed in section 1, candidates can choose six if they wish to be considered for the award of an A level GCE grade and three for the award of an AS grade. The allowable combinations are set out below:

Specification title	Entry code	Assessment units to be taken
AS Mathematics	S2211	C1 C2 M1
AS Mathematics	S2211	C1 C2 S1
A level Mathematics	A2211	C1 C2 C3 C4 M1 M2
A level Mathematics	A2211	C1 C2 C3 C4 S1 S4
A level Mathematics	A2211	C1 C2 C3 C4 M1 S1
AS Further Mathematics	S2331	F1 F2 S1
AS Further Mathematics	S2331	F1 S1 M3
AS Further Mathematics	S2331	F1 S1 S4
AS Further Mathematics	S2331	F1 F2 M1
AS Further Mathematics	S2331	F1 M1 M2
AS Further Mathematics	S2331	F1 M2 S4
AS Further Mathematics	S2331	F1 M2 M3
AS Further Mathematics	S2331	F1 F2 M2
AS Further Mathematics	S2331	F1 F2 S4
AS Further Mathematics	S2331	F1 F2 M3
AS Further Mathematics	S2331	F1 M3 M4
A level Further Mathematics	A2331	F1 F2 F3 M3 M4 S1
A level Further Mathematics	A2331	F1 F2 F3 M3 S1 S4
A level Further Mathematics	A2331	F1 F2 F3 M1 M2 M3
A level Further Mathematics	A2331	F1 F2 F3 M2 M3 S4
A level Further Mathematics	A2331	F1 F2 F3 M2 M3 M4

Units that contribute to an award in AS and A level Mathematics may not also be used for an award in Further Mathematics.

To gain certificates in both Advanced Mathematics and AS Further Mathematics, candidates must use 9 different units.

To gain certificates in both Advanced Mathematics and Advanced Further Mathematics, candidates must use 12 different units.

Units C1, C2, M1, S1 and F1 are assessed at AS standard; all other units are assessed at A2 standard.

Centres entering candidates for AS/A level Further Mathematics should be aware of the following implications of the choice of units for the A level award:

- candidates who have taken a **single** applications strand (either Mechanics or Statistics) for their A level award will have **two AS units** (F1 and either S1 or M1) available for their Further Mathematics award;

- candidates who have taken **both** applications strands (Mechanics and Statistics) for their A level award will have **only one AS unit** (F1) available for their Further Mathematics award.

These allowable combinations are consistent with the requirements of the Subject Criteria for Mathematics (2002):

- AS and A level specifications in Mathematics must address at least one applications area;
- AS Further Mathematics specifications must include at least one unit of pure mathematics;
- A Level Further Mathematics specifications must include at least two units of pure mathematics;
- A level mathematics specifications must include two or three A2 units;
- A level Further Mathematics specifications must include at least three A2 units.

Example of Pathways to AS and A level Mathematics and to AS and A level Further Mathematics

AS	A2	AS FM	FM
C1 C2 S1	C3 C4 M1	F1 M2 S4	F2 F3 M3
C1 C2 M1	C3 C4 S1	F1 M2 M3	F2 F3 S4
C1 C2 M1	C3 C4 M2	F1 S1 F2	F3 M3 M4
			F3 M3 S4
		F1 S1 M3	F2 F3 M4
			F2 F3 S4
		F1 S1 S4	F2 F3 M3
C1 C2 S1	C3 C4 S4	F1 M1 M2	F2 F3 M3
		F1 M1 F2	F3 M2 M3

Application, in writing, must be made to the Council if centres wish to enter candidates for an AS or A level Pure Mathematics award.

THE RELATIONSHIP BETWEEN ASSESSMENT UNITS AND ASSESSMENT OBJECTIVES

TABLE 1 : AS ASSESSMENT WEIGHTINGS

Candidates must take three assessment units to qualify for an AS award. Each of the three units is weighted at 33 1/3% of the award. The allowable combinations of assessment units are set out on page 12.

	Assessment Objectives				
Assessment Unit	AO1 %	AO2 %	AO3 %	AO4 %	AO5 %
C1	30 - 40	35 - 45	5 - 15	5 - 10	0
C2	30 - 40	35 - 45	5 - 15	5 - 10	8 - 15
M1	30 - 40	25 - 30	20 - 30	10 – 20	10 - 15
S1	30 - 40	25 - 30	20 - 30	10 – 20	10 - 15
F1	30 - 40	40 - 50	0	5 - 10	0 - 5

TABLE 2 : A LEVEL ASSESSMENT WEIGHTINGS

Candidates must take six assessment units to qualify for an A level award. Each of the six units is weighted at 16 2/3% of the award. The allowable combinations of assessment units are set out in the previous section.

	Assessment Objectives				
Assessment Unit	AO1 %	AO2 %	AO3 %	AO4 %	AO5 %
C1	30 - 40	35 - 45	5 - 15	5 - 10	0
C2	30 - 40	35 - 45	5 - 15	5 - 10	8 - 15
C3	30 - 40	35 - 45	5 - 15	5 - 10	5 – 10
C4	30 - 40	35 - 45	5 - 15	5 - 10	5 – 10
F1	30 - 40	40 - 50	0	5 - 10	0 - 5
F2	30 - 40	30 - 40	10 - 20	5 - 10	5 – 10
F3	30 - 40	30 - 40	10 - 20	5 - 15	5 – 10
M1	30 - 40	25 - 30	20 - 30	10 – 20	10 - 15
M2	30 - 40	25 - 30	20 - 30	5 - 15	5 – 15
M3	30 – 40	30 - 40	10 - 20	5 - 15	5 – 20
M4	30 - 40	25 - 40	15 - 25	5 - 15	5 – 15
S1	30 - 40	25 - 30	20 - 30	10 – 20	10 - 15
S4	30 - 40	25 - 30	20 - 30	5 - 15	10 - 20

NATURE OF ASSESSMENT UNITS

All assessment units under this specification will be external. All assessment units take the form of an external examination and the duration of each examination is shown below:

- Assessment Unit C1 assesses Module C1 1 hour 30 minutes
- Assessment Unit C2 assesses Module C2 1 hour 30 minutes
- Assessment Unit C3 assesses Module C3 1 hour 30 minutes
- Assessment Unit C4 assesses Module C4 1 hour 30 minutes
- Assessment Unit F1 assesses Module FP1 1 hour 30 minutes
- Assessment Unit F2 assesses Module FP2 1 hour 30 minutes
- Assessment Unit F3 assesses Module FP3 1 hour 30 minutes
- Assessment Unit M1 assesses Module M1 1 hour 30 minutes
- Assessment Unit M2 assesses Module M2 1 hour 30 minutes
- Assessment Unit M3 assesses Module M3 1 hour 30 minutes
- Assessment Unit M4 assesses Module M4 1 hour 30 minutes
- Assessment Unit S1 assesses Module S1 1 hour 30 minutes
- Assessment Unit S4 assesses Module S2 1 hour 30 minutes

THE SEQUENCE, TIMING AND RESITTING OF ASSESSMENT UNITS

In 2005, the first year of certification of AS Mathematics under this specification, assessment units C1, C2, C3, C4, F1, F2, M1, M2, S1 and S4 will be offered in the summer examination series only. No other assessment units will be offered in 2005.

Thereafter, assessment units will be offered as follows:

Candidates may sit the following assessment units in either the winter or summer examination sessions:
C1, C2, C3, C4, F1, F2, M1, M2, S1.

Candidates may sit the following assessment units in the summer examination session only:
F3, M3, M4, S4.

Assessment units may be retaken more than once. The best result in each unit must count towards the final award; other results in these units will be used up. Candidates may, however, retake the whole qualification more than once. The results of individual assessment units, prior to certification, will have a shelf life limited only by the shelf life of this specification.

SYNOPTIC ASSESSMENT

Synoptic assessment in mathematics will address candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the A level course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

Synoptic assessment must form 20% of the total assessment for A level mathematics. The synoptic units in A level mathematics are C3, C4, M1, M2, S1and S4. Candidates may take these units in any order and in any examination series. Candidates will automatically fulfil the synoptic requirements by taking any of the allowable combinations shown on page 12 of this specification.

In papers which address the A2 core content, synoptic assessment requires the use of methods from the AS core content. In papers which address mathematical content outside the core content, synoptic assessment requires the use of methods from the core content and/or methods from earlier stages of the same aspect of mathematics (pure mathematics, mechanics or statistics).

LANGUAGE OF SPECIFICATION AND ASSESSMENT MATERIALS

The specification and associated specimen assessment materials are provided in English.

CANDIDATES WITH PARTICULAR REQUIREMENTS

Details of arrangements for candidates with particular assessment requirements are provided in the Joint Council for General Qualifications GCSE and GCE Regulations and Guidance for Candidates with Special Assessment Needs.

AWARDS AND CERTIFICATION

The AS and the Advanced GCE in Mathematics, Pure Mathematics and Further Mathematics will be awarded on a five-grade scale: A, B, C, D and E. Candidates who fail to reach the minimum standard for a grade E will be recorded as U (unclassified) and will not receive an AS or A level GCE certificate. The results of individual assessment units will be reported.

Where AS certification is not requested, a candidate going on to complete the full A level GCE must nevertheless complete all the modules and take all the assessment units required for the AS award.

CCEA will comply with the grading, awarding and certification requirements of the revised GCE Code of Practice for courses starting in September 2000.

3 SUBJECT CONTENT

This specification builds on the knowledge, understanding and skills established in GCSE Mathematics. The core material for AS, contained in teaching and learning modules C1 and C2, is a subset of the core material for A level; this is completed in modules C3 and C4.

The subject content is organised into thirteen teaching and learning modules. The content of these modules is set out below. For each module the major topics are listed, together with related guidance notes. These notes provide further detail of the content required but **they are not intended to be exhaustive descriptions** of the topics to which they relate.

Within the categories of Pure Mathematics, Mechanics and Statistics, the modules are set out in the normal sequence in which their associated assessment units would be taken. The content of each unit should be read in conjunction with the relevant aims and assessment objectives set out in Section 1 of this specification.

This specification for AS and A level Mathematics requires:

(a) construction and presentation of mathematical arguments through appropriate use of logical deduction and precise statements involving correct use of symbols and appropriate connecting language;
(b) correct understanding and use of mathematical language and grammar in respect of terms such as ‘equals’, ‘identically equals’, ‘therefore’, ‘because’, ‘implies’, ‘is implied by’, ‘necessary’, ‘sufficient’ and notation such as $\therefore$, $\Rightarrow$, $\Leftarrow$ and $\Leftrightarrow$.

In addition, the specification for A level mathematics requires:

(c) methods of proof, including proof by contradiction and disproof by counter-example.

Candidates are expected to make use of clear, precise and appropriate mathematical language in accordance with the requirements of Assessment Objective 2. This will be assessed by the allocation of marks against Assessment Objective 2.

At least one area of the application of mathematics must be addressed by candidates seeking an award in AS or A level Mathematics. For this specification, this means that candidates must study Mechanics, Statistics or both. The application of mathematics must count for at least 30% of the total credit for the qualification.

This specification for A level Mathematics satisfies the requirement to include content, mainly in the area of pure mathematics, to study some aspect of modelling and the application of mathematics. Modelling should be addressed, as appropriate, in the teaching of all modules in the specification. For both AS and A level Mathematics, the knowledge, understanding and skills identified by the respective criteria must attract two-thirds of the total credit for the qualification.

MODULE C1 – AS CORE MATHEMATICS 1

This module covers approximately half of the core content material for AS examinations. The module will be assessed at AS standard and is compulsory for AS and A level GCE Mathematics. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. **Candidates are not permitted to use any calculating aid in the assessment unit for this module.**

Topic		Guidance Notes
1	Laws of indices for all rational indices.	
	Use and manipulation of surds.	Rationalisation of denominators
2	Quadratic functions and their graphs; the discriminant of a quadratic function; completing the square.	Conditions for real and equal roots are included. Excluding relationship between roots and coefficients of a quadratic equation.
	Solution of quadratic equations.	
	Simultaneous equations; analytic solution by substitution, eg of one linear and one quadratic equation.	Linear with two or three unknowns.
	Solution of linear and quadratic inequalities.	Including inequalities reducible to the form $f(x) > 0$, where $f(x)$ is a product of linear factors.
3	Algebraic manipulation of polynomials, including expanding brackets and collecting like terms, factorisation and simple algebraic division.	Division by linear expressions only.
	Use of the Factor Theorem and the Remainder Theorem.	
4	Graphs of functions; sketching curves defined by simple equations.	Knowledge of function notation. Plotting graphs on graph paper will not be required.
	Geometrical interpretation of algebraic solution of equations. Use of intersection points of graphs to solve equations.	
	Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	
5	Equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$.	Including the mid-point of a line segment.
	Conditions for two straight lines to be parallel or perpendicular to each other.	

Topic		**Guidance Notes**
6	The derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a point; the gradient of the tangent as a limit; interpretation as a rate of change; second order derivatives.	Questions on differentiation from first principles will not be set.
	Differentiation of x^n and related sums and differences.	
	Applications of differentiation to gradients, tangents and normals, maxima and minima and stationary points, increasing and decreasing functions.	Including use of second derivative.

MODULE C2 – AS CORE MATHEMATICS 2

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. This module, together with module C1, contains core content material for AS examinations. A knowledge of the content of module C1 will be assumed. The module will be assessed at AS standard and is compulsory for AS and A level GCE Mathematics. The assessment unit for the module is an external examination with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		Guidance Notes
1	Co-ordinate geometry of the circle using the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$, and including use of the following circle properties: • Angle in a semicircle is a right angle; • Perpendicular from centre to a chord bisects the chord; • Perpendicularity of radius and tangent.	Including the form $x^2 + y^2 + 2gx + 2fy + c = 0$
2	Sequences, including those given by a formula for the *n*th term and those generated by a simple relation of the form $x_{n+1} = \mathrm{f}(x_n)$	Behaviour of sequences (convergence, divergence and oscillation) will be required.
	Arithmetic series, including the formula for the sum of the first n terms.	Including the use of the Σ notation formula for the sum of the first n natural numbers.
	The sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$.	
	Binomial expansion of $(1 + x)^n$ for positive integer n; the notations $n!$ and $\binom{n}{r}$.	Cases of $(a + b)^n$ reducible to the form $k(1 + x)^n$ may be examined.
3	Sine and cosine rules and the area of a triangle in the form $\frac{1}{2}ab\sin C$.	
	Radian measure, including use for arc length and area of a sector.	$s = r\theta$; $A = \frac{1}{2}r^2\theta$
	Sine, cosine and tangent functions; their graphs, symmetries and periodicity. Knowledge and use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$.	
	Solution of simple trigonometric equations in a given interval.	For example: $3 - 3\cos\theta - \sin^2\theta = 0$ for $-\pi < \theta \le \pi$

Topic		Guidance Notes
4	$y = a^x$ and its graph.	Knowledge of the effect of simple transformations.
	Laws of logarithms: • $\log_a x + \log_a y = \log_a (xy)$ • $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ • $k \log_a x = \log_a (x^k)$.	Including the change of base rule: $\log_a x = \frac{\log_b x}{\log_b a}$ Excluding natural logarithms.
	The solution of equations of the form $a^x = b$.	Excluding the use of the logarithmic transformation in changing experimental data into straight-line form.
5	Indefinite integration as the reverse of differentiation.	
	Integration of x^n and related sums and differences.	
	Approximation of area under a curve using the trapezium rule.	
	Interpretation of the definite integral as the area under a curve. Evaluation of definite integrals.	Area defined by curve and either axis or between two curves.

MODULE C3 – A2 CORE MATHEMATICS 1

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. This module covers approximately half of the core content material for A level examinations beyond the AS core material contained in modules C1 and C2. A knowledge of the content of modules C1 and C2 will be assumed. The module will be assessed at A2 standard and is compulsory for A level GCE Mathematics. The assessment unit for the module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Guidance Notes**
1	Simplification of rational expressions including factorizing and cancelling, and algebraic division.	Division by non-linear expressions.
	Rational functions; partial fractions with denominators not more complicated than repeated linear terms.	
	The modulus function.	Including $\|x - a\| < b$
	Combinations of simple transformations on the graph of $y = \mathrm{f}(x)$ as represented by $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$.	
2	Parametric equations of curves; conversion between parametric and Cartesian forms.	General properties of conics are excluded.
3	Binomial series for any rational value of n.	
4	Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent.	Use of notation $\sin^{-1}\theta$ etc.
	Knowledge and use of the equivalents of $\sin^2\theta + \cos^2\theta = 1$.	$\sec^2\theta = \tan^2\theta + 1$ $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$ Solution of trigonometric equations in a given interval; for example, $2\sec^2\theta + 5\tan\theta = 5$.
5	The function e^x and its graph.	Knowledge of the effect of simple transformations.
	The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Knowledge of the effect of simple transformations.
	Exponential growth and decay.	Both discrete and continuous growth. For example, the half-life of a radioactive element.

	Topic	**Guidance Notes**
6	Differentiation of e^x, ln$x$, sin$x$, cos$x$, tan$x$ and their sums and differences.	For example, ln3x, lnx^2.
	Differentiation using the product rule, the quotient rule, the chain rule and by the use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$.	For example, e^{3x}, ln(x^2 + 2), sin3x, $\tan^2 x$. Differentiation of sec$x$, cosec$x$ and cot$x$. Including second derivative. Excluding connected rates of change.
	Indefinite integration as the reverse of differentiation; in particular, integration of e^x, $\frac{1}{x}$, sin$x$, cos$x$.	For example: e^{2x}, sin3x, $\frac{2}{3x}$, $\sec^2 \frac{x}{2}$
7	Location of roots of f(x) = 0 by considering changes of sign of f(x) in an interval of x in which f(x) is continuous.	
	Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n-1} = \mathrm{f}\left(x_n\right)$	Including Newton-Raphson method.
	Numerical integration of functions.	Simpson's Rule.

MODULE C4 – A2 CORE MATHEMATICS 2

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. This module, together with modules C1, C2 and C3, contains the core content material for A level examinations. A knowledge of the content of modules C1, C2 and C3 will assumed. The module will be assessed at A2 standard and is compulsory for A level GCE Mathematics. The assessment unit for the module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		Guidance notes
1	Definition of a function; domain and range of functions; composition of functions; inverse functions and their graphs.	Knowledge that $\mathrm{gf}(x) \equiv \mathrm{g}(\mathrm{f}(x))$. $\mathrm{f}^{-1}(x)$.
2	Understanding of the graphs and appropriate restricted domains of secant, cosecant, cotangent, arcsin, arccos and arctan.	
3	Knowledge and use of double angle formulae;	Proofs of these may be required.
	use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$; and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	Proofs of these are not required. Solution of trigonometric equations (excluding general solution) including use of the identities listed.
4	Differentiation of simple functions defined implicitly or parametrically.	Including second derivative.
	Formation of simple differential equations.	
5	Simple cases of integration by substitution and by parts; these methods as the reverse processes of the chain and product rules respectively.	For example: $\int \sin^2 x\,\mathrm{d}x$; $\int \cos^3 x\,\mathrm{d}x$ The relationship with corresponding techniques of differentiation should be understood. The 't' ($t = \tan \theta/2$) substitution is excluded. Integration by parts will not require more than one operation.
	Simple cases of integration using partial fractions.	
	Evaluation of volume of revolution.	Volumes generated by the rotation of the area under a single curve about the x-axis only.
	Analytical solution of simple first order differential equations with separable variables.	

Topic		**Guidance notes**
6	Vectors in two and three dimensions.	Notation: $\overrightarrow{AB}$, $\underline{r}$, **r** Unit vectors: **i**, **j**, **k**. Including column vector notation.
	Magnitude of a vector.	Notation: $\lvert\overrightarrow{AB}\rvert$, $\lvert\underline{r}\rvert$, $\lvert\mathbf{r}\rvert$.
	Algebraic operations of vector addition and multiplication by scalars and their geometrical interpretations.	Excluding questions set in a geometrical context.
	Position vectors; the distance between two points; vector equations of lines.	
	The scalar product; its use for calculating the angle between two lines.	Excluding skew lines. $\mathbf{a}.\mathbf{b} = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$ where θ is the angle between **a** and **b**.

MODULE FP1 – FURTHER PURE MATHEMATICS 1

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. A knowledge of the content of modules C1 and C2 will be assumed. The module will be assessed at AS standard and is compulsory for AS and A level GCE Further Mathematics. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		Guidance Notes
1	Matrices: addition, multiplication, null and unit matrices.	
	Solution of linear equations in 2 and 3 unknowns.	Illustrated by solution of $\mathbf{Ax} = \mathbf{y}$
	Evaluation of inverses of non-singular matrices.	
2	Linear mappings and transformations in the plane.	Represented by matrices and column vectors. To include linear transformations of curves, $f(x, y) = 0$
3	Determinants; implication of the zero value of the determinant of:	Determinants of order 2 and 3 only.
	(i) a simple transformation matrix;	Only order 2.
	(ii) the coefficient matrix of a system of simultaneous linear equations.	Order 2 and 3.
4	Eigenvalues and eigenvectors of 3 × 3 matrices.	Including 2 × 2 matrices. Reduction of symmetrical matrices to diagonal form.
5	Binary operations and groups; period of an element; cyclic groups, isomorphism between groups. Subgroups.	Symmetry groups, permutation groups, groups of 2 × 2 matrices and the group of residue classes mod m are included. Lagrange's theorem (without proof).
6	Further co-ordinate geometry of a circle.	Intersection of circles, equation of common chord, equation of a tangent to a circle.
7	Complex numbers; Cartesian and polar form, modulus, argument, conjugate.	
	Argand diagrams.	
	Sum, difference, product and quotient of two complex numbers.	Representation on an Argand diagram of the sum or difference of two complex numbers may be required.
	Simple loci	For example, $\|z - z_1\| = \|z - z_2\|$; $\|z - a\| = r$.

MODULE FP2 – FURTHER PURE MATHEMATICS 2

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. A knowledge of the content of modules C1, C2, C3, C4 and FP1 will assumed. The module will be assessed at A2 standard and is compulsory for A level GCE Further Mathematics. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Guidance Notes**
1	Partial fractions	To include quadratic factors in the denominator.
2	Summation of finite series. Use of Σr, Σr^2 and Σr^3.	To include arithmetic series, geometric series and telescopic series.
3	Proof by mathematical induction.	
4	General solution of trigonometric equations.	
5	Simple treatment of the co-ordinate geometry of the parabola and ellipse in Cartesian and parametric form. Change of origin without rotation.	Including definition of curves as loci; knowledge of focus, directrix and eccentricity.
6	De Moivre's theorem for general index excluding proof. The n th roots of a complex number. The exponential form of a complex number. Complex roots of simple polynomials with real coefficients.	Including expansions of sin $n\,\theta$ and cos $n\,\theta$ in powers of sin θ and cos θ and the reverse process. Extension to expression for tan $n\,\theta$.
7	Analytical solution of the differential equations $y' + p(x)y = q(x)$ and $ay'' + by' + cy = f(x)$ where a, b and c are constants. Solutions satisfying boundary conditions.	The form of $f(x)$ will be restricted to $k \cos nx$, $k \sin nx$, kx^n or $k\,e^{nx}$, where k is a constant and n is an integer; $f(x)$ will not be a solution of the corresponding homogeneous equation: $ay'' + by' + cy = 0$ The auxiliary equation may have complex roots.

Topic		**Guidance Notes**
8	Maclaurin's theorem.	
	The derivation of the series expansion of $(1 + x)^n$, e^x, $\ln(1 + x)$, $\sin x$, $\cos x$ and $\tan^{-1} x$.	To include the derivation of the series expansions of simple compound functions.
	Simple exercises on, approximations by and simple variations on these expansions.	
	Summation of infinite series using series expansions.	
	$\sin x \approx x$, $\cos x \approx 1 - \frac{1}{2}x^2$, $\tan x \approx x$.	

MODULE FP3 – FURTHER PURE MATHEMATICS 3

In following a course based on this specification students should be encouraged to make appropriate use of 'graphic' calculators and computers as tools by which the learning of mathematics may be enhanced. A knowledge of the content of modules C1, C2, C3, C4, FP1 and FP2 will be assumed. The module will be assessed at A2 standard and is compulsory for A level GCE Further Mathematics. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. The use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

	Topic	**Guidance notes**
1	Differentiation of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$.	
2	Repeated integration by parts. Simple reduction formulae.	Integration by parts will not involve more than three applications.
3	Integration of $\dfrac{1}{a^2+x^2}$, $\dfrac{1}{\sqrt{a^2-x^2}}$.	Use of appropriate substitutions.
4	The hyperbolic and inverse hyperbolic functions; their definitions, graphs, derivatives and integrals.	Including relationships between the six hyperbolic functions and the ln expressions for sinh-1and cosh-1.
5	Cartesian equation of a line.	$\dfrac{x-a}{l}=\dfrac{y-b}{m}=\dfrac{z-c}{n}$ Including skew lines.
6	Vector product.	$\lvert\mathbf{a}\times\mathbf{b}\rvert$ as an area. $\mathbf{a}.\mathbf{b}\times\mathbf{c}$ as a volume. The equation of a line $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$. The vector triple product is excluded.
7	Vector and Cartesian equations of a plane. Lines normal and parallel to a plane; planes normal and parallel to a line; angle between a line and a plane; equation of the line of intersection of two planes.	$\mathbf{n}.\mathbf{r}$ = d and $ax + by + cz = d$.

MODULE M1 – MECHANICS 1

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. Students should be given opportunities to explore practically, contexts relating to the contents of this module. A knowledge of syllabus C1 and C2 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at AS level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Comment**
1	Displacement, velocity and acceleration; displacement-time graphs; velocity-time graphs.	Rectilinear motion only.
	Equations for uniform acceleration.	
	Application of differentiation and integration to problems in kinematics set as a function of time.	Acceleration given as a simple function of t.
2	Force as a localised vector.	
	Magnitude, direction, components and resultants.	
3	Friction.	Modelling assumptions about frictional force. Limiting friction = μN . Angle of friction not required.
4	Equilibrium of a particle.	
5	Moment of a force about a point. The principle of moments.	Parallel forces; couples. Concept of Centre of Gravity. Non-parallel forces.
6	Equilibrium of a rigid body.	Including ladders and rods.
7	Mass and acceleration.	
	Newton's laws of motion to include motion of connected particles.	Where pulleys are involved they will be smooth and fixed.
8	Impulse and momentum.	
	Principle of conservation of linear momentum; direct impact.	Rectilinear motion and direct impacts only (coefficient of restitution not required).

MODULE M2 – MECHANICS 2

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. Students should be given opportunities to explore practically, contexts relating to the contents of this module. A knowledge of syllabus C1, C2, C3, C4 and M1 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at A2 level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Comment**
1	Displacement, velocity, acceleration, force etc as vectors.	Vectors in the form $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$.
2	Integration and differentiation of vectors.	Vectors in the form $\mathrm{f}(t)\mathbf{i} + \mathrm{g}(t)\mathbf{j} + \mathrm{h}(t)\mathbf{k}$.
3	Variable acceleration along a straight line.	Acceleration as a function of time or velocity or displacement. Examples involving constant power may be set.
4	Projectiles.	
	Motion in a vertical plane with constant acceleration, ie under gravity.	Derivation of the standard results for greatest height reached, time of flight, range on a horizontal plane and the equation of the flight-path is required. Examples involving an inclined plane will not be set.
5	Uniform motion in a horizontal circle.	$v = r\omega$, $a = \omega^2 r = v^2/r$. Including banked corners. Excluding sliding/overturning problems.
	Conical pendulum.	
6	Gravitational potential energy (mgh).	
	Kinetic energy.	
	Work done.	Excluding the calculation by integration of work done by a variable force.
	Work-energy principle.	Work done = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ = change of kinetic energy.
	Principle of conservation of mechanical energy.	
7	Power treated as rate of doing work (leading to $P = Fv$) and rate of increase of energy.	Rectilinear motion only.

MODULE M3 – MECHANICS 3

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. Students should be given opportunities to explore practically, contexts relating to the contents of this module. A knowledge of syllabus C1, C2, C3, C4, M1 and M2 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at A2 level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

	Topic	Comment
1	Centre of mass.	Proof of standard results not required.
	System of particles at fixed points.	Including use of symmetry. Excluding use of calculus. Excluding variable density.
	Rods.	
	Rectangular, triangular and circular laminae.	
	Composite laminae.	
	Suspended laminae.	
2	Further particle equilibrium.	Including a particle attached to elastic springs or strings on a rough plane.
3	Resultant velocity.	Graphical or vector component method.
	Relative velocity.	Including problems involving minimum distance or interception, but not course for closest approach.
4	Hooke's Law.	Modelling assumption: elastic limits.
	Elastic springs and strings.	
5	Work and Kinetic energy.	Use of scalar product. Rectilinear motion only.
	Work-energy principle.	Work = $\int_a^b F\,dx$ $= \frac{1}{2}mv_a^2 - \frac{1}{2}mv_b^2$
	Energy stored in an elastic spring or string.	= change in kinetic energy.
	Simple problems involving kinetic energy, gravitational potential energy and elastic potential energy.	
6	Simple harmonic motion.	Knowledge of definition and standard results. Proof of standard results is not required.
	Simple pendulum.	
	Oscillations of a particle attached to the end of an elastic spring or string.	Oscillations will be in the direction of the spring or string.

MODULE M4 – MECHANICS 4

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. Students should be given opportunities to explore practically, contexts relating to the contents of this module. A knowledge of syllabus C1, C2, C3, C4, M1, M2 and M3 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at A2 level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Comment**
1	Centre of mass of laminae and solids. Composite bodies. Suspended bodies. Toppling problems.	Excluding variable density. Including use of calculus. Proof of standard results for solid cone and solid hemisphere only may be required. Table of standard results may be used.
2	Force systems in two dimensions; general resultant of coplanar force systems.	Replacement of system by a single force, by a couple or by a single force acting at a specific point together with a couple.
3	Light pin-jointed frameworks.	Use of Bow's notation is optional. Questions may involve identifying forces as tension or thrust as well as calculating their magnitude.
4	Method of dimensions.	Checking of expressions and equations for dimensional consistency. Derivation of equations connecting physical quantities where a product relationship is assumed.
5	Universal Law of Gravitation; satellite motion.	
6	Further circular motion on banked corners.	Questions may be set on sliding/overturning problems.
7	Motion in a vertical circle.	Proof of standard results may be required.
8	Direct impact of elastic spheres; Newton's law of restitution. Elastic collisions between a smooth sphere and a plane or between smooth spheres.	Questions involving impulsive tensions in strings will not be set.

MODULE S1 – STATISTICS 1

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. A knowledge of syllabus C1 and C2 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at AS level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Candidates should be familiar with methods of presenting data, including frequency tables for ungrouped and grouped data, box plots and stem-and-leaf diagrams. They should also be familiar with mean, mode and medium as summary measures of location of data. Questions that directly test the ability of candidates to construct such tables and diagrams and calculate such measures will not be set, but candidates will be expected to interpret them and draw inferences from them.

	Topic	**Comment**
1	Appreciation of the inherent variability of data.	
	Collection, ordering and presentation of data.	Candidates will be expected to draw inferences about data sets and histograms of varying widths and interpret results at a level beyond that expected at GCSE.
2	Calculation and interpretation of appropriate summary measures of the location and dispersion of data.	Knowledge, understanding and use of mean, median, mode, inter-quartile range, and standard deviation. Computation of standard deviation (and of mean) should be by calculator. Candidates should know which key to use when calculating the standard deviation of a data set. Candidates will be expected to draw inferences about data sets and interpret results at a level beyond that expected at GCSE.
3	Sample space: events, mutually exclusive and exhaustive events. Classical and limiting relative frequency definitions of probability.	$P(A)=\dfrac{n(\mathrm{A})}{n}$, where event A is a subset of all n equally likely outcomes. $P(A)=\lim\limits_{m\to\infty}\dfrac{f_A}{m}$, where f_A is the frequency of event A in m trials.
4	Addition Law; Multiplication Law; statistical dependence and independence.	Calculation of combined probabilities of up to three events using appropriate diagrams. Including conditional probability.
5	Probability functions, mean, variance and standard deviation.	Calculation of probabilities such as $\mathrm{P}(a \le X \le b)$ and of mean and variance for simple cases. Knowledge and use of expressions for $\mathrm{E}(a + bX)$ and $\mathrm{Var}(a + bX)$.

Topic		**Comment**
6	Discrete probability distributions: uniform, binomial, Poisson.	Knowledge and use of the probability functions and the expressions for the mean and variance, but not their derivation. Knowledge of assumptions for binomial and Poisson distribution is required. Ability to use the exponential function to calculate probabilities for a Poisson distribution is required. Use of recurrence formulae will not be required.
7	Continuous probability distribution; probability density function f ; mean, variance and standard deviation.	Calculation of probabilities such as $P(a < X < b)$ and of mean and variance for simple cases. The pdf will be given as a simple function of x. Knowledge and use of expressions for $E(a + bx)$ and $Var(a + bX)$.
8	Normal distribution; linear transformation of a Normal variable; the standard Normal distribution.	Applications only. Use of N(0,1) tables.

MODULE S2 – STATISTICS 2

In following a course based on this specification students should be encouraged to make appropriate use of graphic calculators and computers as tools by which the learning of mathematics may be enhanced. A knowledge of syllabus C1, C2, C3, C4 and S1 will be assumed. Modelling and the application of mathematics as referred to at the start of section 3 will be assessed in this module. The module will be assessed at A2 level and is optional. The assessment unit for this module is an external examination, with a maximum of 75 raw marks, the duration of which is 1 hour 30 minutes. Use of a 'graphic' or 'scientific' calculator will be permitted in the assessment unit for this module.

Topic		**Comment**
1	Expectation algebra.	Knowledge and use of expressions for E $(aX + bY)$ and Var$(aX + bY)$, where X and Y are independent. (Proofs will not be required.)
2	Linear combination of independent Normal variates.	Including E $(\Sigma X_i /n)$ and Var $(\Sigma X_i /n)$, where X_i are independent observations of X.
3	Simple random sampling.	Use of random number tables and generation of pseudo random numbers from a calculator or computer.
	Central Limit Theorem.	Use in cases when n >30.
	Point estimation of population mean and variance.	Use of $S^2 = \dfrac{\sum (X_i - \overline{X})^2}{n-1}$ as an unbiased estimator of σ^2.
	Standard error of mean.	
	Confidence intervals for population mean.	
4	Hypothesis testing.	Knowledge of null and alternative hypotheses, significance level, critical region, one-tailed and two-tailed.
	Normal test for the mean.	Use of tests: Sample mean (large sample) $\dfrac{\overline{X} - \mu}{S/\sqrt{n}}$ is approximately distributed as N(0,1).
	t-test for the mean.	Sample mean (small sample from a normal population) $\dfrac{\overline{X} - \mu}{S/\sqrt{n}}$ has a student's t distribution with n-1 degrees of freedom. Including paired comparisons.

Topic		**Comment**
5	Bivariate distributions; scatter diagrams; product-moment correlation.	Qualitative understanding of hypotheses testing using r ($H_0 : \rho = 0$).
6	Linear regression.	Knowledge of model assumptions. Knowledge of explanatory (independent) and response (dependent) variables, excluding transformations of variables. Equation of line of best fit by least squares method. Difficulties with extrapolation. Derivation of the formulae will not be required.

4 GRADE DESCRIPTIONS

ADVANCED GCE IN MATHEMATICS

The following grade descriptions indicate the level of attainment characteristic of the given grade at A level. They give a general indication of the required learning outcomes at each specified grade. The descriptions should be interpreted in relation to the content outlined in the specification; they are not designed to define that content. The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performances in others.

Grade A

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can often devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are sometimes noticed and corrected.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They correctly refer results from calculations using the model to the original situation; they give sensible interpretations of their results in the context of the original realistic situation. They make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer the results of calculations back to the given context and usually make sensible comments or predictions. They can distil the essential mathematical information from extended pieces of prose having mathematical content. They can comment meaningfully on the mathematical information.

Candidates make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Grade C

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and sometimes proceed logically through extended arguments or proofs. When confronted with unstructured problems they sometimes devise and implement an effective and efficient solution strategy. They occasionally notice and correct errors in their calculations.

Candidates recall or recognise most of the standard models that are needed and usually select appropriate ones to represent a variety of situations in the real world. They often correctly refer results from calculations using the model to the original situation, they sometimes give sensible interpretations of their results in the context of the original realistic situation. They sometimes make intelligent comments on the modelling assumptions and possible refinements to the model.

Candidates comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer the results of calculations back to the given context and sometimes make sensible comments or predictions. They distil much of the essential mathematical information from extended pieces of prose having mathematical content. They give some useful comments on this mathematical information.

Candidates usually make appropriate and efficient use of contemporary calculator technology and other permitted resources, and are sometimes aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Grade E

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the standard models that are needed and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes correctly refer results from calculations using the model to the original situation; they try to interpret their results in the context of the original realistic situation.

Candidates sometimes comprehend or understand the meaning of translations in mathematics of common realistic contexts. They sometimes correctly refer the results of calculations back to the given context and attempt to give comments or predictions. They distil some of the essential mathematical information from extended pieces of prose having mathematical content. They attempt to comment on this mathematical information.

Candidates often make appropriate and efficient use of contemporary calculator technology and other permitted resources. They often present results to an appropriate degree of accuracy.

5 RESOURCE LIST

The following list is an indication of books and other resources which teachers and students may find useful in teaching and studying a course based on this specification. It is not intended to be a list of prescribed texts, nor is it intended to be an exhaustive list of all available resources. The Council cannot accept responsibility for the availability of these resources.

Title	**Author**	**Publisher and ISBN Number**
Pure Mathematics		
Understanding Pure Mathematics	A J Sadler & D W S Thorning	Oxford Press 0-19-914243-2
Introducing Pure Mathematics (Second Edition)	Robert Smedley and Garry Wiseman	Oxford University Press 0-19-914803-2
Pure Mathematics	Joyce S Batty Book 1 Book 2	Schofield & Sons Ltd 0-7217-2356-X 0-7217-2357-8
Core Maths for Advanced GCE	L Bostock & S Chandler	Nelson Thornes 0-7487-5509-8
Pure Mathematics	Martin Brown, Rigby and Riley	Nelson Thornes 0-7487-3558-5
Further Pure Mathematics	Brian and Mark Gaulter	Oxford University Press 0-19-914735-3
Advanced Mathematics: A Pure Course	M and P Perkins	Collins Educational 0-7135-2821-4
Mechanics		
Understanding Mechanics	A J Sadler & D W S Thorning	Oxford University Press 0-19-914675-6
Mechanics for A-Level	Bostock and Chandler	Nelson Thornes 0-7487-2596-2
Complete Advanced Level Mathematics: Mechanics	Adams, Haighton and Trim	Nelson Thornes 0-7487-3559-3
Mechanics and Statistics		
Mathematics Mechanics and Probability	L Bostock & S Chandler	Stanley Thornes Ltd 0-8595-0141-8
Advanced Mathematics: An Applied Course	M and P Perkins	Collins Educational 0-0032-2270-5

Title	**Author**	**Publisher and ISBN Number**
Statistics		
Introducing Statistics (Second Edition)	Graham Upton and Ian Cook	Oxford University Press 0-19-914801-5
Understanding Statistics	Ian Cook and Graham Upton	Oxford University Press 0-19-914391-9
Advanced Statistics	Michael Hugill	Collins Educational 0-00-322215-2
A concise course in Advanced Level Statistics (4th Edition)	Crawshaw & Chambers	Nelson Thornes 0-7487-5475-X
Complete Advanced Level Mathematics: Statistics	McLennon, McGill and Migliorini	Nelson Thornes 0-7487-3560-7
Statistics: A Course for ‘A’ Level Mathematics	M E M Jones Book 1 Book 2	Schofield and Sims Ltd 0-7217-2360-8 0-7217-2361-6

6 LIST OF FORMULAE WHICH WILL BE GIVEN

PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$
Area of curved surface of cone = $\pi r \times$ slant height

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Arithmetic Series

$u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\left[2a+(n-1)d\right]$$

Geometric Series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ \dots \ + \binom{n}{r}a^{n-r}b^r + \ \dots \ + b^n \qquad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ \dots \ + \frac{n(n-1)\dots(n-r+1)}{1.2\dots r}x^r + \dots \qquad (|x|<1, n \in \mathbb{R})$$

Logarithms and exponentials

$\mathrm{e}^{x\ln a} = a^x$

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Complex Numbers

$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots$

Maclaurin's Series

$\mathrm{f}(x) = \mathrm{f}(0) + x\mathrm{f}'(0) + \dfrac{x^2}{2!}\mathrm{f}''(0) + \ldots + \dfrac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$

$\mathrm{e}^x = \exp(x) = 1 + x + \dfrac{x^2}{2!} + \ldots + \dfrac{x^r}{r!} + \ldots$ for all x

$\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots + (-1)^{r+1}\dfrac{x^r}{r} + \ldots \quad (-1 < x \le 1)$

$\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots + (-1)^r\dfrac{x^{2r+1}}{(2r+1)!} + \ldots$ for all x

$\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + (-1)^r\dfrac{x^{2r}}{(2r)!} + \ldots$ for all x

$\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \ldots + (-1)^r\dfrac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \le x \le 1)$

$\sinh x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \ldots + \dfrac{x^{2r+1}}{(2r+1)!} + \ldots$ for all x

$\cosh x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \ldots + \dfrac{x^{2r}}{(2r)!} + \ldots$ for all x

$\tanh^{-1} x = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ldots + \dfrac{x^{2r+1}}{2r+1} + \ldots \quad (-1 < x < 1)$

Hyperbolic Functions

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\} \quad (x \geq 1)$

$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2 + 1}\right\}$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Conics

	Ellipse	Parabola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$
Parametric Form	$(a\cos\theta, b\sin\theta)$	$(at^2, 2at)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	$e = 1$
Foci	$(\pm ae, 0)$	$(a, 0)$
Directrices	$x = \pm\frac{a}{e}$	$x = -a$
Asymptotes	none	none

Trigonometry

In the triangle ABC: $a^2 = b^2 + c^2 - 2\,bc\cos A$

Trigonometric Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + {}^{1}/{}_{2})\pi)$

$\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

$\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$

$\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

$\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \mathbf{b}.(\mathbf{c} \times \mathbf{a}) = \mathbf{c}.(\mathbf{a} \times \mathbf{b})$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector **b** has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \; (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$n_1x + n_2y + n_3z + d = 0$ where $d = -\,\mathbf{a.n}$.

The plane through non-collinear points A, B and C has vector equation

$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\,\mathbf{a} + \lambda\,\mathbf{b} + \mu\,\mathbf{c}$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $\mathbf{r} = \mathbf{a} + s\,\mathbf{b} + t\,\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

f(x)	f′(x)
$\tan kx$	$k\sec^2 kx$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\,\mathrm{g}(x) - \mathrm{f}(x)\,\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\text{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\dfrac{1}{1+x^2}$

Integration

(+ constant; a > 0 where relevant)

f(x)	$\int \text{f}(x)\text{d}x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\text{cosec}\,x$	$-\ln\lvert\text{cosec}\,x + \cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)$, $(\lvert x\rvert < a)$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$

$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 - a^2}\right\},\ (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a+x}{a-x}\right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right),\ (\lvert x \rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert,\ (\lvert x \rvert > a)$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

NUMERICAL MATHEMATICS

Numerical integration

The trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Simpson's rule: $\int_a^b y\,dx \approx \frac{1}{3}\mathrm{h}\{(\mathrm{y}_0 + \mathrm{y}_\mathrm{n}) + 4(\mathrm{y}_1 + \mathrm{y}_3 + .. + \mathrm{y}_{\mathrm{n}-1}) + 2(\mathrm{y}_2 + \mathrm{y}_4 + ..\mathrm{y}_{\mathrm{n}-2})\}$,

where $h = \frac{b-a}{n}$ and n is even

Numerical Solution of Equations

The Newton-Raphson iteration for solving f(x) = 0: $x_{n+1} = x_\mathrm{n} - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

Centres of Mass

For uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r: $\frac{3}{8}r$ from centre

Hemispherical shell, radius r: $\frac{1}{2}r$ from centre

Circular arc, radius r, angle at centre 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Sector of circle, radius r, angle at centre 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\dfrac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h: $\dfrac{1}{3}h$ above the base on the line from centre of base to vertex

Moments of Inertia

For uniform bodies of mass m

Thin rod, length $2l$, about perpendicular axis through centre: $\dfrac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\dfrac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\dfrac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\dfrac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\dfrac{1}{3}m(a^2+b^2)$

Hoop or cylindrical shell of radius r about axis: mr^2

Hoop of radius r about a diameter: $\dfrac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis: $\dfrac{1}{2}mr^2$

Disc of radius r about a diameter: $\dfrac{1}{4}mr^2$

Solid sphere, radius r, about diameter: $\dfrac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\dfrac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x-y plane)

Universal law of gravitation

Force $= \dfrac{Gm_1m_2}{d^2}$

PROBABILITY & STATISTICS

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B|A)$

$P(A|B) = \dfrac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$

Bayes' Theorem: $P(A_j|B) = \dfrac{P(A_j)P(B \mid A_j)}{\sum P(A_i)P(B \mid A_i)}$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $E(X) = \mu = \Sigma_i x_i p_i$
Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma_i (x_i - \mu)^2 p_i = \Sigma_i x_i^2 p_i - \mu^2$
For a function $g(X)$: $E(g(X)) = \Sigma_i g(x_i) p_i$

Standard discrete distributions:

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \dfrac{\lambda^x}{x!}$	λ	λ

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $E(X) = \mu = \int x \ f(x)dx$
Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x - \mu)^2 \ f(x)\,dx = \int x^2 \ f(x)dx - \mu^2$
For a function $g(X)$: $E(g(X)) = \int g(x) \ f(x)dx$
Cumulative distribution function: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$

Standard continuous distributions

Distribution of X	P.D.F.	Mean	Variance
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2

Expectation algebra

Covariance: $\mathrm{Cov}(X, Y) = \mathrm{E}((X - \mu_X)(Y - \mu_Y)) = \mathrm{E}(XY) - \mu_X\mu_Y$
$\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) \pm 2ab\mathrm{Cov}\,(X, Y)$

For independent random variables X and Y
$\mathrm{E}(XY) = \mathrm{E}(X)\mathrm{E}(Y)$, $\mathrm{Var}(aX \pm bY) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y)$

Sampling distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\bar{X}$ is an unbiased estimator of μ, with $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \frac{\Sigma(X_i - \bar{X})^2}{n-1}$

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$$

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$ (also valid in matched-pairs situations)

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \frac{x}{n}$, then

$\mathrm{E}(Y) = p$ and $\mathrm{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then

$$\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}} \sim t_{n_x+n_y-2} \text{ where } S_p^2 = \frac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\{\Sigma(x_i-\bar{x})^2\}\{\Sigma(y_i-\bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

7 LIST OF FORMULAE WHICH WILL NOT BE GIVEN

The following formulae are those which candidates are expected to remember and will not be included in formulae books.

Quadratic equations	**Formula first needed for module**
$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	C1
Laws of logarithms	
$\log_a x + \log_a y \equiv \log_a (xy)$	C2
$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$	C2
$k \log_a x \equiv \log_a \left(x^k\right)$	C2
Trigonometry	
In the triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	C2
area $= \dfrac{1}{2} ab \sin C$	C2
$\cos^2 A + \sin^2 A \equiv 1$	C2
$\sec^2 A \equiv 1 + \tan^2 A$	C3
$\text{cosec}^2 A \equiv 1 + \cot^2 A$	C3
$\sin 2A \equiv 2 \sin A \cos A$	C4
$\cos 2A \equiv \cos^2 A - \sin^2 A$	C4
$\tan 2A \equiv \dfrac{2 \tan A}{1 - \tan^2 A}$	C4

Differentiation

function	derivative	
x^n	nx^{n-1}	C1
$\sin kx$	$k \cos kx$	C3
$\cos kx$	$-k \sin kx$	C3
e^{kx}	ke^{kx}	C3
$\ln x$	$\dfrac{1}{x}$	C3
$f(x) + g(x)$	$f'(x) + g'(x)$	C1
$f(x)\, g(x)$	$f'(x)\, g(x) + f(x)\, g'(x)$	C3
$f(g(x))$	$f'(g(x))\, g'(x)$	C3

Integration

function	integral	
x^n	$\frac{1}{n+1}x^{n+1} + c,\ n \neq -1$	C2
$\cos kx$	$\frac{1}{k}\sin kx + c$	C3
$\sin kx$	$-\frac{1}{k}\cos kx + c$	C3
e^{kx}	$\frac{1}{k}e^{kx} + c$	C3
$\frac{1}{x}$	$\ln\lvert x\rvert + c,\ x \neq 0$	C3
$f'(x) + g'(x)$	$f(x) + g(x) + c$	C2
$f'(g(x))\,g'(x)$	$f(g(x)) + c$	C4

Area

area under a curve $= \int_a^b y\,dx \ (y \geq 0)$ C2

Volume

volume formed by revolution through 2π radians about x - *axis* $= \pi\int_a^b y^2\,dx$ C4

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$ C4

MECHANICS FORMULAE

Straight Line Motion

Constant acceleration

$v = u + at$ M1

$v^2 = u^2 + 2as$ M1

$s = ut + \frac{1}{2}at^2$ M1

$s = \frac{1}{2}(u + v)t$ M1

Variable acceleration

acceleration $= \frac{dv}{dt} = v\frac{dv}{dx}$ M2

Elastic Strings/Springs

Tension $= \frac{\lambda x}{l}$ M3

Momentum and Energy

Momentum $= mv$ M1

Gravitational potential energy $= mgh$ M2

Elastic potential energy $= \frac{\lambda x^2}{2l}$ M3

Kinetic energy $= \frac{1}{2}mv^2$ M2

Work $= \int_A^B F\,dx = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$ M3

Power $= Fv$ M2

Simple Harmonic Motion

$\dfrac{d^2x}{dt^2} = -\omega^2 x$	M3
$v^2 = \omega^2(a^2 - x^2)$	M3
$T = \dfrac{2\pi}{\omega}$	M3
$x = a \sin(\omega t + \varepsilon)$ or $x = A \sin \omega t + B \cos \omega t$	M3
$v = a\omega \cos(\omega t + \varepsilon)$	M3

Vectors

Expressions for velocity and acceleration in 2 dimensions.

$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$	M2
$\mathbf{v} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j}$	M2
$\mathbf{a} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j}$ where $\dot{x} = \dfrac{dx}{dt}, \ddot{x} = \dfrac{d^2x}{dt^2}, \dot{y} = \dfrac{dy}{dt}, \ddot{y} = \dfrac{d^2y}{dt^2}$	M2
Scalar product $\mathbf{a} \cdot \mathbf{b} = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \cos \theta$	M2

STATISTICAL FORMULAE

Statistical Measures

For a sample of size n we define:

Mean	$\bar{x} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$	S1
Variance	$S^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \dfrac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2$	S1

If n is the sample size and f_i the frequency of occurrence of the value x_i in the sample (so that $n = \Sigma f_i$) then:

Mean	$\bar{x} = \dfrac{1}{n} \sum f_i x_i$	S1
Variance	$s^2 = \dfrac{1}{n} \sum f_i (x_i - \bar{x})^2 = \dfrac{1}{n} \sum f_i x_i^2 - \bar{x}^2$	S1

Expectation Algebra

$E(a + bx) = a + bE(x)$	$\text{Var}(a + bx) = b^2 \text{Var}(x)$	S1
$E(\sum_i x_i) = nE(x)$	$\text{Var}(\sum_i x_i) = n \text{Var}(x)$	S1

Test of significance

(i) Sample mean (large sample)

$\dfrac{\overline{X}-\mu}{S/\sqrt{n}}$ is approximately distributed as N(0, 1) S2

(ii) Sample mean (small sample from a normal population)

$\dfrac{\overline{X}-\mu}{S/\sqrt{n}}$ has a students' t-distribution with n–1 degrees of freedom S2

8 GLOSSARY OF MATHEMATICAL NOTATION

1 **Set Notation**

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
$\{x: \ldots\}$	the set of all x such that …
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathcal{E}$	the universal set
A'	the complement of the set A
$\mathbf{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbf{Z}$	the set of integers, $\{0, \pm1, \pm2, \pm3, \ldots\}$
$\mathbf{Z}^+$	the set of positive integers, $\{1, 2, 3 \ldots\}$
$\mathbf{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbf{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbf{Z},\ q \in \mathbf{Z}^+\right\}$
$\mathbf{Q}^+$	the set of positive rational numbers, $\{x \in \mathbf{Q} : x > 0\}$
$\mathbf{Q}^+_0$	the set of positive rational numbers and zero, $\{x \in \mathbf{Q} : x \geq 0\}$
$\mathbf{R}$	the set of real numbers
$\mathbf{R}^+$	the set of positive real numbers, $\{x \in \mathbf{R} : x > 0\}$
$\mathbf{R}^+_0$	the set of positive real numbers and zero, $\{x \in \mathbf{R} : x \geq 0\}$
$\mathbf{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the Cartesian product of sets A and B ie $A \times B = \{(a, b): a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection

$[a, b]$	the closed interval $\{x \in \mathbf{R}: a \le x \le b\}$
$[a, b)$, $[a, b[$	the interval $\{x \in \mathbf{R}: a \le x < b\}$
$(a, b]$, $]a, b]$	the interval $\{x \in \mathbf{R}: a < x \le b\}$
(a, b), $]a, b[$	the open interval $\{x \in \mathbf{R}: a < x < b\}$
$y\,R\,x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 **Miscellaneous Symbols**

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\le$, $\ngtr$	is less than or equal to, is not greater than
$>$	is greater than
$\ge$ $\nless$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 **Operations**

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a/b	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbf{Z}^+$ $\frac{n(n-1)\ \ldots(n-r+1)}{r!}$ for $n \in \mathbf{Q}$

4 **Functions**

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \to y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
$\mathrm{g} \circ \mathrm{f}$, gf	the composite function of f and g which is defined by $(\mathrm{g} \circ \mathrm{f})(x)$ or $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
$\Delta x, \delta x$	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x

$f'(x), f''(x), \ldots, f^{(n)}(x)$	the first, second, … , *n*th derivatives of f(*x*) with respect to *x*
$\int y\,dx$	the indefinite integral of *y* with respect to *x*
$\int_a^b y\,\mathrm{d}x$	the definite integral of *y* with respect to *x* between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of *V* with respect to *x*
$\dot{x}, \ddot{x}, \ldots\ldots$	the first, second, … derivatives of *x* with respect to time

5 **Exponential and Logarithmic Functions**

e	base of natural logarithms
e^x, exp *x*	exponential function of *x*
$\log_a x$	logarithm to the base *a* of *x*
$\ln x$, $\log_e x$	natural logarithm of *x*
$\lg x$, $\log_{10} x$	logarithm of *x* to base 10

6 **Circular and Hyperbolic Functions**

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$ OR arcsin, arccos, arctan, arccosec, arcsec, arccot	the inverse circular functions
sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, cosech^{-1}, sech^{-1}, $\coth^{-1}$ OR ar(c)sinh, ar(c)cosh, ar(c)tanh ar(c)cosech, ar(c)sech, ar(c)coth	the inverse hyperbolic functions

7 Complex Numbers

i, j	square root of –1
z	a complex number, $z = x + \mathrm{i}y$ $= r(\cos\theta + \mathrm{i}\sin\theta)$
Re z	the real part of z, Re z , = x
Im z	the imaginary part of z, Im z, = y
$\lvert z\rvert$	the modulus of z, $\lvert z\rvert = \sqrt{(x^2 + y^2)}$
arg z	the argument of z, arg z = θ, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z, x - iy

8 Matrices

M	a matrix **M**
$\mathbf{M}^{-1}$	the inverse of the matrix **M**
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix **M**
det **M** or |**M**|	the determinant of the square matrix **M**

9 Vectors

a	the vector **a**
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of **a**
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
|**a**|, a	the magnitude of **a**
$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
a . b	the scalar product of **a** and **b**
a × **b**	the vector product of **a** and **b**

10 Probability and Statistics

A, B, C, etc	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
X, Y, R, etc	random variables
x, y, r, etc	values of the random variables X, Y, R, etc
$x_1, x_2, \ldots$	observations
$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
$\mathrm{f}(x), \mathrm{g}(x), \ldots$	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x), \mathrm{G}(x), \ldots$	the value of the (cumulative) distribution function $\mathrm{P}(X \le x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}[\mathrm{g}(X)]$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}\,(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values 0, 1, 2, …
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$, m	sample mean

S^2, $\hat{\sigma}^2$	unbiased estimate of population variance from a sample, $S^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution N(0, 1)
Φ	corresponding cumulative distribution function
ρ	product moment correlation coefficient for a population
r	product moment correlation coefficient for a sample
Cov (X, Y)	covariance of *X* and *Y*

APPENDIX

Opportunities for developing and generating evidence for assessing key skills

The following table signposts and exemplifies the types of opportunity for developing and generating evidence for assessing key skills that may arise during an AS or Advanced GCE in Mathematics and Further Mathematics. The opportunities are referenced to Section B of the relevant key skills specifications at Level 3. The subject exemplifications illustrate typical opportunities which may arise during the normal teaching and learning process. These are only a small selection of such opportunities and are not part of the key skills specifications themselves. It is for teachers and students to decide which pieces of work, if any, to use to develop and assess key skills.

Key Skill: Application of Number

Key Skills Specification Part B Reference		Subject Exemplification
Activity	**Evidence**	
N3.1 Plan, and interpret information from two different types of sources, including a large data set.	• Plan how to obtain and use the information required to meet the purpose of your activity; • Obtain the relevant information; and • Choose appropriate methods for obtaining the results you need and justify your choice.	*AS Module S1* *Appreciation of the inherent variability of data* *Collection, ordering and presentation of data.*
N3.2 Carry out multi-stage calculations to do with: a amounts and sizes; b scales and proportions; c handling statistics; d rearranging and using formulae. You should work with a large data set on at least one occasion.	• Carry out calculations to appropriate levels of accuracy, clearly showing your methods; and • Check methods and results to help ensure errors are found and corrected.	*Carry out calculations related to:* *a AS Module M1 Magnitude, direction, components and resultants of vectors.* *b A2 Module C3 Exponential growth and decay.* *c AS Module S1 Calculation and interpretation of appropriate summary measures of the location and dispersion of data for a large data set.* *d A2 Module M2 Projectiles. Motion in a vertical plane with a constant accelaration, ie under gravity.*

Key Skills Specification Part B Reference Activity	Evidence	Subject Exemplification
N3.3 Interpret results of your calculations, present your findings and justify your methods. You must use at least one graph, one chart and one diagram.	• Select appropriate methods of presentation and justify your choice; • Present your findings effectively; and • Explain how the results of your calculations relate to the purpose of your activity.	*AS Module S1* *Collection, ordering and presentation of data*

Key Skill: Communication

Key Skills Specification Part B Reference		Subject Exemplification
Activity	**Evidence**	
C3.1a Contribute to a group discussion about a complex subject.	• Make clear and relevant contributions in a way that suits your purpose and situation; • Listen and respond sensitively to others, and develop points and ideas; and • Create opportunities for others to contribute when appropriate.	*AS Module S1;* *A2 Module S2.* *Opportunities may arise for a teaching group to discuss the use/abuse of statistics to support particular viewpoints. Candidates may be given the opportunity to research an area of interest and give a presentation of a written report on their findings to the rest of the teaching group.* *Opportunities arising in the context of other fields of study, for example physics, biology or geography, where use may be made of mathematical techniques outlined in modules of this specification.*
C3.1b Make a presentation about a complex subject, using at least **one** image to illustrate complex points.	• Speak clearly and adapt your style of presentation to suit your purpose, subject, audience and situation; • Structure what you say so that the sequence of information and ideas may be easily followed; and • Use a range of techniques to engage the audience, including effective use of images.	
C3.2 Read and synthesise information from **two** extended documents about a complex subject. One of these documents should include at least **one** image.	• Select and read material that contains the information you need; • Identify accurately, and compare, the lines of reasoning and main points from texts and images; and • Synthesise the key information in a form that is relevant to your purpose.	
C3.3 Write **two** different types of documents about complex subjects. One piece of writing should be an extended document and include at least **one** image.	• Select and use a form and style of writing that is appropriate to your purpose and complex subject matter; • Organise relevant information clearly and coherently, using specialist vocabulary when appropriate; and • Ensure your text is legible and your spelling, grammar and punctuation are accurate, so your meaning is clear.	

Key Skill: Information Technology

Key Skills Specification Part B Reference Activity	Evidence	Subject Exemplification
IT3.1 Plan, and use different sources to search for, and select, information required for two different purposes.	• Plan how to obtain and use the information required to meet the purpose of your activity; • Choose appropriate sources and techniques for finding information and carry out effective searches; and • Make selections based on judgements of relevance and quality.	*Application of Assessment Objective AO5:* *Use of contemporary calculator technology and other permitted resources (such as Formulae booklets or statistical tables) accurately and efficiently.* *Understanding of when not to use such technology, and its limitations.*
IT3.2 Explore, develop and exchange information and derive new information to meet two different purposes.	• Enter and bring together information in a consistent form, using automated routines where appropriate; • Create and use appropriate structures and procedures to explore and develop information and derive new information; and • Use effective methods of exchanging information to support your purpose.	*Search for and select relevant formulae from the examination Formulae Sheet and from textbooks to solve problems.* *Use of graphic calculator or tables for standard deviation and for transformations*
IT3.3 Present information from two different sources for two different purposes and audiences. Your work must include at least one example of text, one example of images and one example of numbers.	• Develop the structure and content of your presentation using the views of others, where appropriate, to guide refinements; • Present information effectively, using a format and style that suits your purpose and audience; and • Ensure your work is accurate and makes sense.	*See above.*

Key Skill: Working with Others

Key Skills Specification Part B Reference Activity	Evidence	Subject Exemplification
WO3.1 Plan complex work with others, agreeing objectives, responsibilities and working arrangements.	• Agree realistic objectives for working together and what needs to be done to achieve them; • Exchange information based on appropriate evidence, to help agree responsibilities; and • Agree suitable working arrangements with those involved.	*The modelling aspects which permeate mechanics and statistics and which are present in any in-context question in pure mathematics.* *Opportunities given to explore practically, contexts relating to the content of the mechanics modules.* *Refinement of models stating assumptions made.* *Opportunities arising in the context of other fields of study, for example physics, biology or geography, where use may be made of mathematical techniques outlined in modules of this specification.*
WO3.2 Seeking to establish and maintain co-operative working relationships over an extended period of time, agreeing changes to achieve agreed objectives.	• Organise and carry out tasks so you can be effective and efficient in meeting your responsibilities and produce the quality of work required; • Seek to establish and maintain co-operative working relationships, agreeing ways to overcome any difficulties; and • Exchange accurate information on progress of work, agreeing changes where necessary to achieve these objectives.	
WO3.3 Review work with others and agree ways of improving collaborative work in the future.	• Agree the extent to which work with others has been successful and the objectives have been met; • Identify factors that have influenced the outcome; and • Agree ways of improving work with others in the future.	*See above.*

Key Skill: Improving own Learning and Performance

<table>
<tr><th colspan="2">Key Skills Specification Part B Reference</th><th rowspan="2">Subject Exemplification</th></tr>
<tr><th>Activity</th><th>Evidence</th></tr>
<tr><td>LP3.1 Agree targets and plan how these will be met over an extended period of time, using support from appropriate people.</td><td>• Seek information on ways to achieve what you want to do, and identify factors that might affect your plans;
• Use this information to agree realistic targets with appropriate people; and
• Plan how you will effectively manage your time and use of support to meet targets, including alternative action for overcoming possible difficulties.</td><td rowspan="2">Planning revision and using an appropriate study skills program to set and achieve goals.

Identifying how much progress can be made through a problem using skills already acquired; knowing when to ask for help.

Identifying the hierarchy of skills required and understanding the development of a topic; using this information to improve performance.

Mechanics
2-D motion leading to 3-D motion leading to motion on a inclined plane.</td></tr>
<tr><td>LP3.2 Take responsibility for your learning by using your plan, and seeking feedback and support from relevant sources, to help meet targets.

Improve your performance by:

• Studying a complex subject;
• Learning through a complex practical activity;
• Further study or practical activity that involves independent learning.</td><td>• Prioritise action and manage your time effectively to complete tasks, revising your plan as necessary;
• Seek and actively use feedback and support from relevant sources to help you meet targets; and
• Select and use different ways of learning, to improve your performance, adapting approaches to meet new demands.</td></tr>
<tr><td>LP3.3 Review progress on two occasions and establish evidence of achievements, including how you have used learning from other tasks to meet new demands</td><td>• Provide information on the quality of your learning and performance, including factors that have affected the outcome;
• Identify targets you have met, seeking information from relevant sources to establish evidence of your achievements; and
. Exchange views with appropriate people to agree ways to further improve your performance.</td><td>See above.</td></tr>
</table>

Key Skill: Problem Solving

Key Skills Specification Part B Reference		Subject Exemplification
Activity	**Evidence**	
PS3.1 Explore a complex problem, come up with three options for solving it and justify the option selected for taking forward.	• Explore the problem, accurately analysing its features, and agree with others on how to show success in solving it; • Select and use a variety of methods to come up with different ways of tackling the problem; and • Compare the main features of each option, including risk factors, and justify the option you select to take forward.	*Application of differentiation to max/min/stationary point problems in context.* *Setting up differential equations from a context.* *The modelling aspects which permeate mechanics and statistics and which are present in any in-context question in pure mathematics.* *Opportunities given to explore practically, contexts relating to the content of the mechanics modules.* *Refinement of models stating assumptions made.* *Opportunities arising in the context of other fields of study, for example physics, biology or geography, where use may be made of mathematical techniques outlined in modules of this specification.*
PS3.2 Plan and implement at least one option for solving the problem, review progress and revise your approach as necessary.	• Plan on how to carry out your chosen option and obtain agreement to go ahead from an appropriate person; • Implement your plan, effectively using support and feedback from others; and • Review progress towards solving the problem and revise your approach as necessary.	
PS3.3 Apply agreed methods to check if the problem has been solved, describe the results and review your approach to problem solving.	• Agree with an appropriate person, methods to check if the problem has been solved; • Apply these methods accurately, draw conclusions and fully describe the results; and • Review your approach to problem solving, including whether alternative methods and options might have proved more effective.	*See above.*